Bulgarian Journal of Physics vol. **51** (2024) 130–136

A Simple Equation of State Parameter and its Complex Consequences in Cosmology

F. Robin, C. Sivakumar*

Department of Physics, Maharaja's college, Ernakulam, Kerala, India

*Corresponding author E-mail: sivakumar@maharajas.ac.in

Received: 10 February 2023, *Revised: 26 November 2023*

doi: https://doi.org/10.55318/bgjp.2024.51.2.130

Abstract. An effective equation of state parameter for the cosmic fluid of radiation, matter and dark energy of the form: $\omega_{\text{eff}} = -[1 + (z/(\rho R))\rho_d]/3$, where z is a constant is shown a beautiful expression for representing our flat Universe after its shift to the phase of dark era that is believed to have occurred about 5 billion years back. The model shows how complicated the Universe becomes by the emergence of dark energy and how it gradually decays. It also finds the range of size of the present Universe.

KEY WORDS: Standard model of cosmology; Flat Universe; Effective equation of state parameter; Dark energy; Friedman equations.

1 Introduction

There are a number of observations especially the recent ones of the cosmic microwave background radiation and luminosity-redshift data of Type Ia supernovae which suggest that universe is flat and has an accelerated expansion. The Friedmann model, which is based on the mathematically simple assumptions of spatial homogeneity and isotropy which is expressed through the spatially symmetric Friedmann-Robertson-Walker metric gives a reliable account of the history of the universe from the very early times. These observations suggest that a large fraction, of the energy density of the universe has negative pressure, called dark energy.

The Friedman equations of Standard Cosmology [1] obtained by reducing Einstein's gravitational field equations using Cosmological Principle and Weyl postulate are

$$\frac{\dot{R}^2}{R^2} + \frac{kc^2}{R^2} = \frac{8\pi G\rho}{3}\,, \tag{1}$$

$$\frac{\dot{R}^2}{R^2} + \frac{kc^2}{R^2} = -\frac{8\pi GP}{c^2} - 2\frac{\ddot{R}}{R}\,, \tag{2}$$

 1310–0157 © 2024 Union of the Physicists in Bulgaria (UPB)

where R is the scale factor of Universe and $\frac{\dot{R}}{R} = H$ is the Hubble parameter; k is the curvature parameter which can take any of $+1, 0, -1$; k is $+1$ for closed Universe, 0 for flat Universe and -1 for open Universe; G is the Newton's universal constant of gravitation, P is the uniform pressure and ρ the uniform mass density of the Universe; ρ includes the density ρ_r of radiation, ρ_m of matter and ρ_d of dark energy which are respectively 0%, 32% and 68% approximately according to the present knowledge [2]. c is the speed of light in vacuum. Equations (1) and (2) together mean that

$$\frac{\ddot{R}}{R} = -\frac{4\pi G\rho}{3}(1 + 3\omega_{\text{eff}})\,, \tag{3}$$

where $\omega_{\text{eff}} = \frac{P}{\rho c^2}$ is the effective equation of state parameter of the cosmic fluid that comprises radiation, matter and dark energy. Universe has decelerated expansion if $\omega_{\text{eff}} > -1/3$ and accelerated expansion if $\omega_{\text{eff}} < -1/3$. Planck 2018 results [2] say that our universe is flat and it results in

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G\rho}{3}\,, \tag{4}$$

$$\frac{\dot{R}^2}{R^2} = -\frac{8\pi GP}{c^2} - 2\frac{\ddot{R}}{R}\,. \tag{5}$$

The results of Supernova Cosmology Project (SCP) establishes accelerated expansion of the Universe for the past 5 billion years [3,4]. So, presently it is not radiation or matter that matters since for radiation, ω is $1/3$ and for matter, ω is 0 [5]; we need some form of dark energy such that ω_{eff} is less than $-1/3$ to cause the universe accelerate (the simplest form of dark energy is cosmological constant (cc) for which ω is -1 [6,7]). Also, we have the first law of thermodynamics

$$dU + PdV = dU + \omega_{\text{eff}}\rho c^2 dV = 0\,, \tag{6}$$

where $U \propto R^3\rho$ is the energy of the Universe and $V \propto R^3$ is the volume. This on integration gives, $\rho_r \propto R^{-4}$, $\rho_m \propto R^{-3}$, $\rho_{cc} \propto R^0$.

In Section 2, we describe our model with an effective equation of state parameter. Section 3 discusses the relevant features of the model and its effect on the dynamics of the universe and finally Section 4 contains the conclusions.

2 The Model

The cosmic fluid has radiation, matter and dark energy of which we neglect radiation since its effect lasted only for a small time compared to the age of the Universe, which is approximately 13.8 billion years [2]. Therefore ω_{eff}, inspired by a few reported works [8] is for the composite of matter and dark energy

$$\omega_{\text{eff}} = \frac{P_m + P_d}{(\rho_m + \rho_d)c^2}\,. \tag{7}$$

It is reasonable to assume that the equation of state parameter for dark energy that presently governs the dynamics of the Universe was its minimum -1 at the shift to the dark era, 5 billion years back and increasing afterwards to attain its maximum $-1/3$ finally to save the universe from spending more and more energy for higher and higher accelerations. Let us write ω_{eff}, which has to be a dimensionless number in terms of the scale factor R of the Universe and the percentage of dark energy in the universe which produces acceleration, as it is logically valid.

We postulate the following form for the effective equation of state parameter for the evolution of universe in its dark era

$$\omega_{\text{eff}} = -\frac{1}{3}\Big[1 + \Big(\frac{z}{\rho R}\Big)\rho_d\Big] \tag{8}$$

with z a constant.

3 Discussion of the Model and Results

It is surprisingly interesting to note that Eq. (8) fits perfectly into the formula proposed by Dragan Slavkov Hajdukovic of the PH division CERN [9]

$$\omega_d = -\frac{1}{3}\Big[\frac{\rho_m}{\rho_d} + 1 + \Big(\frac{3}{4R}\Big)\Big(\frac{L_c^3}{L_p^2}\Big)\Big]\,. \tag{9}$$

L_c is the Compton wavelength of pion and L_p the Planck length. Using the relation between pressure and density, one gets

$$\rho\omega_{\text{eff}} = \omega_m\rho_m + \omega_d\rho_d = \omega_d\rho_d\,. \tag{10}$$

Using Eq. (9) and the condition $\frac{\rho_m+\rho_d}{\rho} = 1$, we arrive at

$$\omega_{\text{eff}} = -\frac{1}{3}\Big[1 + \Big(\frac{z}{\rho R}\Big)\rho_d\Big] \tag{11}$$

with $z = \frac{3}{4}\Big(\frac{L_c^3}{L_p^2}\Big)$. Numerically $z = 2.107 \times 10^{27}$ m. It is noted that the order of z is that of the size of the observable universe. But the actual order of size of the universe must be different. Inflation says that it is at least 3×10^{23} times the size of observable universe. L. Susskind's challenge makes it $10^{10^{10^{122}}}$ times; it is 250 times the size of the observable Universe as a lower estimate [10–15].

It can be even smaller than the observable Universe if it is curved [16]. We use Eqs. (10) and (11) to get for the initial, mid and final state of the dark era:

1. When $\omega_d = -\frac{3}{3}$, we have $\frac{\rho_d}{\rho} = -1\omega_{\text{eff}}$ and

$$\omega_{\text{eff}} = \frac{(-\frac{3}{3})R}{3R - z} \,. \tag{12}$$

2. When $\omega_d = -\frac{2}{3}$, we have $\frac{\rho_d}{\rho} = -1.5\omega_{\text{eff}}$ and

$$\omega_{\text{eff}} = \frac{(-\frac{2}{3})R}{2R - z} \,. \tag{13}$$

3. When $\omega_d = -\frac{1}{3}$, we have $\frac{\rho_d}{\rho} = -3\omega_{\text{eff}}$ and

$$\omega_{\text{eff}} = \frac{(-\frac{1}{3})R}{1R - z} \,. \tag{14}$$

The seemingly simple ω_{eff} but results in a very complicated evolution of the cosmological parameters; Universe seems not that simple. To show how complex the situation is with the emergence of dark energy in the universe, let us calculate the density.

1. When $\omega_d = -\frac{3}{3}$,

$$\frac{\rho_m}{\rho_d} = \frac{D_i R_i^3}{R^3} \,; \quad \frac{\rho_d}{\rho} = \frac{\rho_d}{\rho_m + \rho_d} = \frac{1}{1 + \frac{D_i R_i^3}{R^3}} \,, \tag{15}$$

where D_i is a constant. Integration of Eq. (6) (using wolframalpha calculator) from R_i to R, $R - R_i$ sufficiently small, then gives

$$\frac{\rho}{\rho_i} = r_1^{-n_1} r_2^{-n_2} r_3^{-n_3} r_4^{-n_4} r_5^{-n_5} \,, \tag{16}$$

where

$$r_1 = \frac{R}{R_i} \,; \qquad n_1 = 2 \,. \tag{17}$$

If we use $D_i R_i^3 = a_i$, then

$$r_2 = \frac{a_i^{1/3} + R}{a_i^{1/3} + R_i} \,, \qquad n_2 = \frac{z}{3a_i^{1/3}} \,, \tag{18}$$

$$r_3 = \frac{\sqrt{3} - i + 2iR/a_i^{1/3}}{\sqrt{3} - i + 2iR_i/a_i^{1/3}} \qquad n_3 = \frac{zi\sqrt{3}}{6a_i^{1/3}} \,, \tag{19}$$

$$r_4 = \frac{\sqrt{3} + i - 2iR_i/a_i^{1/3}}{\sqrt{3} + i - 2iR/a_i^{1/3}} \qquad n_4 = \frac{zi\sqrt{3}}{6a_i^{1/3}} \tag{20}$$

$$r_5 = \frac{a_i^{2/3} - a_i^{1/3}R + R^2}{a_i^{2/3} - a_i^{1/3}R_i + R_i^2} \qquad n_5 = \frac{z}{6a_i^{1/3}} \tag{21}$$

$(\tan^{-1} y = (\frac{i}{2}) \ln \frac{1-iy}{1+iy}$ with $i = \sqrt{-1}$ has been used.

The Hubble parameter $H = \sqrt{\frac{8\pi G\rho}{3}}$ by Eq. (4) would be highly complicated.

2. When $\omega_d = -\frac{2}{3}$,

$$\frac{\rho_d}{\rho} = \frac{1}{1 + \frac{D_j R_j^2}{R^2}} \,. \tag{22}$$

And around $-\frac{2}{3}$, density evolves according to

$$\frac{\rho}{\rho_j} = f_1^{-m_1} f_2^{-m_2} f_3^{-m_3} \,, \tag{23}$$

where

$$f_1 = \frac{R}{R_j} \,, \qquad m_1 = 2 \,. \tag{24}$$

Using $D_j R_j^2 = a_j$, we have

$$f_2 = \frac{\sqrt{a_j} - iR}{\sqrt{a_j} - iR_j} \,, \qquad m_2 = \frac{iz}{2\sqrt{a_j}} \,, \tag{25}$$

$$f_3 = \frac{\sqrt{a_j} + iR_j}{\sqrt{a_j} + iR} \,, \qquad m_3 = \frac{1z}{2\sqrt{a_j}} \,. \tag{26}$$

3. When $\omega_d = -\frac{1}{3}$,

$$\frac{\rho_d}{\rho} = \frac{1}{1 + \frac{D_k R_k}{R}} \tag{27}$$

and when the universe is very large, for the initial scale factor R_k to the neighbouring scale factor R,

$$\frac{\rho}{\rho_k} = c_1^{s_1} c_2^{s_2} \,. \tag{28}$$

With $D_k R_k = a_k$, we have

$$c_1 = \frac{a_k + R_k}{a_k + R} \,, \qquad s_1 = \frac{z}{a_k} \,, \tag{29}$$

$$c_2 = \frac{R}{R_k} \,, \qquad s_2 = (\frac{z}{a_k}) - 2 \,. \tag{30}$$

Matter to dark energy density ratio for various ω_d is illustrated in the following Table 1:

Table 1. Matter-Dark energy density ratio

ω_d	ρ_m/ρ_d
$-3/3$	$\propto R^{-3}$
$-2/3$	$\propto R^{-2}$
$-1/3$	$\propto R^{-1}$

If the present scale factor R_p is 1×10^{27} m, the present effective equation of state parameter $\omega_{\text{eff}p} = -0.810$ using $\frac{\rho_{dp}}{\rho_p} = 0.68$. This then by Eq. (10) has $\omega_{dp} = -1.191$ which is less than -1.0 and cannot be considered. This means that the size of the Universe is more than 1×10^{27} m if the postulate is true. It cannot be as the inflationary picture or the Susskind challenge, as these lead to practically zero ω_{dp}. $R_p = 250 \times 10^{27}$ gives $\omega_{\text{eff}p} = -0.335$ and $\omega_{dp} = -0.493$ which is in agreement with the model but it is not less than -0.55 as required by the more or less accurate experiments [17]. But if $1.375 \times 10^{27} < R_p < 11.637 \times 10^{27}$, then $-1.000 < \omega_{dp} < -0.550$, which is the desired. Note that all these cases of universes are presently accelerating by Eq. (3).

Table 2. Equation of state parameter

R_p $(\times 10^{27})$	$\omega_{\text{eff}p}$	ω_{dp}
1	-0.810	-1.191
1.375	-0.680	-1.000
11.637	-0.374	-0.550
250	-0.335	-0.493

4 Conclusions

Effective equation of state parameter is not much studied but can be a better alternative for the description of the dynamics of the Universe, it seems in the light of the results it can produce.

The dark energy of the model is not the cosmological constant but one that decays with the size of the Universe. At the shift to the accelerating phase, the dark energy equation of state parameter is -1 and it decreases in its modulus with the expansion of the Universe. This is positive about the model which saves the Universe from spending more and more energy for producing higher and higher accelerations unlike the Standard model in which the acceleration has an exponential increase. Before the shift, density is $\rho = \rho_m$ which is CR^{-3} (C is a constant) since $\omega_{\text{eff}}\rho = 0$ and scale factor $R = (\frac{4\pi C}{3})(1 - \cos\zeta)$, a solution of Friedman equation.

The Universe, with dark energy is very complex and the model elucidates the complexity of evolution of mass density of the Universe and hence the other parameters, if the postulate is true. At the shift, the density given by Eq. (16) is really very complicated because of the inverse cubic $\frac{\rho_m}{\rho_d}$ of R. As the Universe grows, the complexity is gradually reduced to reach its minimum when the effective equation of state parameter is $-1/3$. The present density may be evolving as Eq. (23) as presently, the dark energy equation of state parameter could be near -0.666 and hence the effective equation of state parameter -0.453.

Finally, the model is able to predict a small enough range for the actual size of the universe over which there still exists a confusion. The size of the universe has to be between 1.375×10^{27} and 11.637×10^{27} for consistency with the model and experiments. Note, the current scale factor of the universe is approximately z if the effective equation of state parameter is -0.559.

References

[1] C. Misner, K.S. Thorne, J.A. Wheeler (1973) "*Gravitation*". W.H. Freeman, New York.

[2] N. Aghanim et al. (2020) *Astron. & Astrophys.* **641** id.A6, 67 pp.

[3] S. Perlmutter et al. (1999) *Astrophys. J.* **517** 565.

[4] A.G. Riess et al. (1998) *Astron. J.* **116** 1009.

[5] A. Friedman (1999) *Gen. Relat. Gravit.* **31** 1991.

[6] P.J.E. Peebles, B. Ratra (2003) *Rev. Mod. Phys.* **75** 559.

[7] K.S.Adhav, P.R. Agrawal, R.R. Saraogi (2016) *Bulg. J. Phys.* **43** 171.

[8] J. Ponce de Leon (2002) *Class. Quantum Grav.* **29** 135009.

[9] D.S. Hajdukovic (2009) arXiv.0711.0439 v4 [physics.gen-ph].

[10] C. Amsler et al. (2008) *Phys. Lett. B* **667** 1.

[11] C. Lineweaver, J.M. Hand Davis (2005) *Sci. American* **292** 24.

[12] A.H. Guth (1980) *Phys. Rev. D* **23** 347.

[13] A.H. Guth (1990) "*The Inflationary Universe:The Quest for a New Theory of Cosmic Origins*". Basic Books, New York.

[14] D.N. Page (2007) *JCAP* **0701** 004.

[15] M. Vardanyan, R. Trotta, J. Silk (2011) *MNRS Lett.* **413** L91.

[16] P. Bielewicz, A.J. Banday, K.M. Gorski (2013) arxiv:1303.4004v1.

[17] M. Garnavich et al. (2011) *Astrophys. J.* **509** 74.