Bulgarian Journal of Physics vol. **51** (2024) 185–210

About the Multiverse in the Three-Dimensional Quantum Vacuum with Variable Energy Density

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Received: 22 May 2024
doi: https://doi.org/10.55318/bgjp.2024.51.2.185

Abstract. A model of multiverse in a three-dimensional quantum vacuum characterized by a variable energy density is proposed, where a relationship exists between the evolutionary possibilities of the generic universe and the corresponding fluctuations of the quantum vacuum energy density. A peculiar form of Wheeler-DeWitt equation, intended as an equation of background, in the picture of a "quantum Boltzmann statistics", is considered, which implies the generation of a cyclic multiverse where the evolutionary behaviour of each branch is fixed by the specific quantum vacuum energy density fluctuations characterizing that region of the landscape. A quantum field theory for the wave function of the multiverse in the curved minisuperspace formed by the scale factor and the scalar field that represents the energy-matter content of the universe, is developed. Finally, a suggestive interpretation of cosmological constant and dark matter is explored.

1 Introduction

Many worlds interpretation of quantum mechanics, eternal inflationary model as well as superstring theory lead to the idea that our universe can be considered only an element in a vast set of universes, namely to the idea of a multiverse which is extended at the infinite in the future (and for instance in the past), does not have boundaries and exhibits amazingly variegate properties. In these approaches, the concept of multiverse has the merit to throw new light in facing important issues of today theoretical physics such as the cosmological constant problem or the measurement problem in quantum theory [1–9]. Nonetheless, according to several authors, these multiverse scenarios, despite constituting a change in paradigm, seem to be more speculative ideas than real scientific theories because of the impossibility of being experimentally falsified.

On the other hand, it exists another lesser-known multiverse scenario, which seems to introduce new possibilities as regards the issue of the empirical test of this kind of approaches. It is represented by the idea that there is an interaction between the component universes in the multiverse landscape. By virtue of the interaction between the different universes, important implications on the global structure of the universes arise in the form of a collective behaviour among the universes of the multiverse, which may have an observable influence on the properties of our universe [10]. In this picture, as a result of the interactions among the different elements of the multiverse, a landscape structure is generated where the universes are created with different effective values of the cosmological constant and with different content of fields and particles and, here, quantum tunneling transitions can occur between different universal states which give rise to new bubbles with the corresponding value of their vacuum states.

As regards the existence of universes with different values of the cosmological constant, moreover, one can suppose that there is a relaxing to small values, as a consequence of the fact that the value of the cosmological constant is characterized by a diminishing during the post-inflationary phase. For example, it was shown that, in the picture of an universe born in a metastable false vacuum state corresponding to the local minimum of a canonical scalar field which describes the current stage of accelerated expansion of the universe (and which can be identified with the density of the dark energy), a mechanism can be provided where the cosmological constant Λ can have a very great value resulting from the quantum field theory calculations in the early universe in the inflationary era, $\Lambda \simeq \Lambda_{\mathrm{QFT}}$, while then it can later be quickly reduced to the very, very small values of today universe, as a consequence of reaction of the universe to energy changes during the evolution phases from the time epoch of exponential decay to the late-time epoch characterized by inverse-power law form [11]. On the other hand, a model of multiverse as an ensemble of universes with various cosmological constants has been developed, where the most likely cosmological constant is a sum of exponentially suppressed contributions, i.e. small and positive, without resorting to anthropic arguments, and this result implies the consideration of a temperature of the multiverse which turns out to be less than Planck scale [12].

In this paper, our aim is to explore the process of the nucleation within one single element of the multiverse and the evolutionary possibilities of each element of the multiverse landscape by considering a peculiar form of Wheeler-DeWitt equation, intended as an equation of background, in a picture where a supra-universal background exists which can influence the properties of each region. In our approach, the fundamental background ruling the proprieties of the different regions of the multiverse is associated with a three-dimensional (3D) dynamic quantum vacuum (DQV) characterized by a variable energy density, introduced by the author in several recent works [13–23].

The paper is structured as follows. In Section 2, we explore the foundations and the preliminary concepts of our approach of an interacting multiverse in the 3D DQV characterized by a variable energy density. In Section 3, we make some considerations as regards the average cosmological constant of the parallel universes and its corresponding evolutionary possibilities. In Section 4, we develop the mathematical formalism of the multiverse of the 3D DQV, by starting from a generalized Wheeler-DeWitt equation and derive the consequences as regards the behaviour of each micro-universe and their interactions. In Section 5, we explore the implications as regards galactic dynamics and the interpretation of dark matter. In Section 6, we summarize the main results and perspectives of the model.

2 The Interacting Multiverse in the Three-Dimensional Quantum Vacuum Background: Foundations and Preliminary Concepts

Models of eternal inflation predict that, in the "Eternal Inflation" regime, namely in regions of the parameter space where the quantum fluctuations dominate over classical evolution, universes are constantly being created due to these quantum fluctuations with different cosmological constants and possibly different physical laws in a never ending process. In order to face the embarrassing issue of the smallness of the cosmological constant, the multiverse emerges as the most natural path that can be followed and, in this regard, different kinds of proposals exist, such as dynamical mechanisms [24–27], statistical treatments [28, 29] or various rather radical attempts that even include anthropic considerations (see, for example [30]). However, all these approaches of the multiverse can be considered valid as long as we do not reach planckian temperature or energy density in various universes that require quantum gravity.

In order to portray a picture of the multiverse which can be considered valid in the different domains of energy, in our model we consider that the different physical entities existing in the universe, both at the macroscopic domain and at the microscopic regime, are generated by the different states of a fundamental variable energy density of a 3D DQV intended as the ultimate background of processes. In this model, the geometry of the 3D DQV, by virtue of quantum vacuum energy density fluctuations, turns out to be characterized by a breakdown of the Heisenberg relations at the Planck scale, which leads to the generation of a scale where Compton wavelength and Schwarzschild radius are unified. As a consequence of the existence of this scale where microscopic regime of elementary particles and macroscopic domain of black holes are unified, we assume that each universe of the multiverse can be associated to specific energy density fluctuations of the 3D DQV and, in this picture, the different values of the fundamental quantum vacuum energy density fluctuations imply that each universe turns out to be characterized by a different cosmological constant, by a different content of particles and fields and thus by a different total energy.

In this model, by following [13–23] the appearance of a material particle of mass can be seen as the result of a variable energy density of the 3D quantum vacuum defined as

$$\Delta\rho_{qvE} \equiv \rho_{pE} - \rho_{qvE} = \frac{mc^2}{V}\,, \tag{1}$$

where

$$\rho_{pE} = \frac{m_p c^2}{l_p^3} \tag{2}$$

can be interpreted as the "ground state" of the 3D quantum vacuum, m_p being Planck's mass, c the light speed and l_p Planck's length and ρ_{qvE} is the energy density of the vacuum in the centre of the particle. The variable energy density of the 3D DQV (1) emerges as the fundamental property which provides a unifying treatment of gravity, dark energy and dark matter, which arise here as emergent structures, forms of collective organizations assuming the form of specific excited states of the DQV corresponding to specific fluctuations of the quantum vacuum energy density. Compatibly with the results obtained in [31–33], here the fundamental origin of the curvature of space-time characteristic of general relativity is represented by a quantized metric of the 3D quantum vacuum condensate given by relation

$$d\hat{s}^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu\,, \tag{3}$$

where, the coefficients of the metric are quantum operators which are defined (in polar coordinates) as

$$\begin{aligned} &\hat{g}_{00} = -1 + \hat{h}_{00}, \quad \hat{g}_{11} = 1 + \hat{h}_{11}, \quad \hat{g}_{22} = r^2(1 + \hat{h}_{22}), \\ &\hat{g}_{33} = r^2 \sin^2\vartheta(1 + \hat{h}_{33}), \quad \hat{g}_{\mu\nu} = \hat{h}_{\mu\nu} \quad \text{for} \quad \mu \neq \nu. \end{aligned} \tag{4}$$

and, at the order $O\left(r^2\right)$, one has

$$\begin{aligned} &\left\langle \hat{h}_{\mu\nu} \right\rangle = 0 \\ \text{except} \quad &\left\langle \hat{h}_{00} \right\rangle = \frac{8\pi G}{3}\left(\frac{\Delta\rho_{qvE}}{c^2} + \frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right) r^2 \\ \text{and} \quad &\left\langle \hat{h}_{11} \right\rangle = \frac{8\pi G}{3}\left(-\frac{\Delta\rho_{qvE}}{2c^2} + \frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right) r^2\,. \end{aligned} \tag{5}$$

In equations (5), $\hbar$ is Planck's reduced constant, G is Newton's gravitational constant, $\Delta\rho_{qvE}^{DE}$ are specific fluctuations of the quantum vacuum energy density generating the action of the dark energy density ρ_{DE} and $\left\langle \hat{h}_{\mu\nu} \right\rangle$ stands for $\langle\Psi|\,\hat{h}_{\mu\nu}\,|\rangle$ namely are the average values of these operators in the macroscopic states

$$\Psi\left[\omega\right] = \int D_F \Phi e^{iS} \omega\left[\Phi\right], \tag{6}$$

where

$$D_F\left[\Phi_A, \Phi_B\right] \approx \Psi\left[\Phi_A\right] \Psi\left[\Phi_B\right]^* \approx \mathrm{e}^{i(S[\Phi_A]-S[\Phi_B])} \tag{7}$$

is the "decoherence" functional measuring the quantum interference between two virtual histories A and B of the universe and ω can be considered as a "filter" function that selects which fine-grained histories $\Phi(x)$ are associated to the same superposition with their relative phases [13–23]. By starting from the quantized metric (3), whose coefficients are defined by relations (4) and (5), in this model it is possible to derive the quantum Einstein equations

$$\hat{G}_{\mu\nu} = \frac{8\pi G}{c^4}\hat{T}_{\mu\nu}\,, \tag{8}$$

where the right-hand side, as a consequence of the variable quantum vacuum energy density, can be seen as a stochastic inhomogeneous field which is able to avoid the overwhelmingly large prediction for cosmic acceleration.

In this picture, a crucial concept is that the curvature of space-time associated with a dark energy density emerges as a mathematical value of the changes of the 3D quantum vacuum energy density. By starting from equations (3)-(5), opportune mathematical manipulations as well as comparison with the Friedmann equations, lead to the following fundamental equation regarding the dark energy density

$$\rho_{DE} \cong \frac{35Gc^2}{2\pi\hbar^4}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6 . \tag{9}$$

Equation (9) physically means that, in the 3D DQV model, dark energy does not exist as a primary physical reality but exists only as a peculiar manifestation, a collective emerging phenomenon, of specific quantum vacuum energy density fluctuations. In other words, the interplay of opportune specific fluctuations of the energy density of the 3D DQV represents the real origin and cause of the accelerated expansion of space. In this way, the consideration of dark energy density as an emergent phenomenon from a more fundamental interplay of quantum vacuum energy density fluctuations, as expressed by equation (9), can resolve the cosmological constant problem in an elegant way without the need of a self-tuning, reproducing the observations that the dark energy density turns out to be 123 orders of magnitude lower than the Planck energy density (2).

Now, in the multiverse determined by the variable energy density of the 3D DQV, the crucial point is that one deals with different universes which turn out to be characterized by different values of the cosmological constant as a consequence of the different kinds of fluctuations $\Delta\rho_{qvE}^{DE}$ of the quantum vacuum energy density that mimic the action of dark energy. It is the peculiar energetic fluctuations of the 3D DQV which fix the different possibilities of evolution of the parallel universes and thus here also expansion and contraction can be seen as evolutionary phases which are determined and fixed by the different specific behaviours of the elementary energetic fluctuations of the 3D DQV. This implies in turn that the total energy of each universe is directly determined by these peculiar fluctuations of the quantum vacuum energy density. In other words, we can say that the different kinds of quantum vacuum energy density fluctuations are the fundamental elements that give origin to different universes, to the different sectors of the landscape of the multiverse and their corresponding behaviours and evolutions.

Moreover, in the different universes of the multiverse, the variable energy density of the 3D DQV can be associated to a deformation of the geometry of the background in the sense that, at the Planck scale, one deals with a breakdown of the Heisenberg uncertainty relations that can be replaced by the following generalized uncertainty relations

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left(1+\beta l_p^2 \frac{\Delta\rho_{qvE}^2 V^2}{\hbar^2 c^2}\right), \tag{10}$$

where the parameter β is a fluctuating parameter which expresses the fact that here space-time fluctuations fix the minimal length scale only on average. The generalized uncertainty relations (10), describing the deformation of the geometry of the 3D DQV, lead to a suggestive unification of the microscopic regime of elementary particles and the macroscopic domain of black holes, in the sense that a scale unfolds where Compton wavelength and Schwarzschild radius are unified, which is expressed by relation

$$R'_C = R'_S = \sqrt{\left(\frac{\beta\hbar c}{\Delta\rho_{qvE} V}\right)^2 + \left(\beta l_p^2 \frac{\Delta\rho_{qvE} V}{\hbar c}\right)^2} \tag{11}$$

and thus microscopic systems and macroscopic regime can be seen as emergent physical patterns which derive from more elementary objects [34].

3 About the Cosmological Constant of the Parallel Universes in the Three-Dimensional Quantum Vacuum

In order to develop a mathematical formalism for the multiverse in the 3D DQV, which allows us to derive the properties of the parallel universes as well their evolutionary possibilities, from the fundamental 3D DQV intended as a supra-universal background, we start by using de-Sitter coordinates and thus consider that each universe has a volume given by the size of the de-Sitter horizon, that is connected to the specific fluctuations $\Delta\rho_{qvE}^{DEs}$ mimicking the action of dark energy on the basis of relation

$$V_s^{10} = 4\pi\sqrt{3}\frac{m_{Pl}^3}{\left[\frac{35Gc^2}{2\pi\hbar^4}\left(\frac{1}{c^2}\Delta\rho_{qvE}^{DEs}\right)^6\right]^{3/2}} \tag{12}$$

and thus has a total energy

$$\mathcal{E}_s = 4\pi\sqrt{3}\frac{m_{Pl}^3}{\left[\frac{35Gc^2}{2\pi\hbar^4}\left(\frac{V_s}{c^2}\Delta\rho_{qvE}^{DEs}\right)^6\right]^{1/2}}, \tag{13}$$

where, in our conventions, the de-Sitter horizon is at

$$R_s = \sqrt{\frac{3}{\frac{35Gc^2}{2\pi\hbar^4}\left(\frac{V_s}{c^2}\Delta\rho_{qvE}^{DEs}\right)^6}}\, m_{Pl}\,.$$

By considering now a micro-canonical ensemble of universes with a given N total number of universes, if n_s is the number of the universes with energy $\mathcal{E}_s$, the energy and the entropy of this micro-canonical system are

$$E = \sum_{s=1}^{N} n_s \mathcal{E}_s\,, \tag{14}$$

$$S = N\ln N - \sum_{s=1}^{N} n_s \ln n_s\,.. \tag{15}$$

If an universe with energy $\mathcal{E}_s$ changes its energy to $\mathcal{E}_s'$, the energy of the ensemble of universes changes by

$$\Delta E = \mathcal{E}_s' - \mathcal{E}_s \tag{16}$$

and the entropy by

$$\Delta S \cong -\ln\left(\frac{n_{s'}}{n_s}\right)\,. \tag{17}$$

As a result, the temperature of the ensemble of universes at equilibrium is

$$T = \left(\frac{\Delta E}{\Delta S}\right)_N = -\frac{\mathcal{E}_s' - \mathcal{E}_s}{\ln\left(\frac{n_{s'}}{n_s}\right)} \tag{18}$$

which leads to the probability density

$$\rho_s = A\exp\left[\frac{\ln\left(\frac{n_{s'}}{n_s}\right)}{\mathcal{E}_s' - \mathcal{E}_s}\mathcal{E}_s\right], \tag{19}$$

which provides a canonical ensemble of universes with Boltzmann-like behaviour. Hence, one gets the following result regarding the average cosmological constant of the universes

$$\langle \Lambda \rangle = \sum_s \frac{\Lambda_s \exp\left(\frac{\ln\left(\frac{n_{s'}}{n_s}\right)}{\mathcal{E}'_s - \mathcal{E}_s}\mathcal{E}_s\right)}{\sum_i \exp\left(\frac{\ln\left(\frac{n_{s'}}{n_s}\right)}{\mathcal{E}'_i - \mathcal{E}_i}\mathcal{E}_i\right)} . \tag{20}$$

Relation (20) implies that as long as $\frac{\ln\left(\frac{n_{s'}}{n_s}\right)}{\mathcal{E}'_s - \mathcal{E}_s}\mathcal{E}_s \gg 1$, which is to be expected from large universes and low temperature, the contribution to the cosmological constant is expected to be exponentially small.

Since the multiverse does not have a conserved number of universes, it can be described by a grand potential with a chemical potential, instead of the number of universes, as a state variable. Moreover, in our model we consider that the amazingly variegate features of the different sectors of the landscape of the multiverse can be associated to the so-called "quantum Boltzmann statistics", or "Infinite statistics", which can be considered a necessary ingredient to provide a physical characterization to the entropy of a quantum foam and to reproduce the non-local features of the background. The "quantum Boltzmann statistics are expressed by the q deformation of the commutation relations of the oscillators, expressed by the following relation:

$$\hat{b}_k \hat{b}_l^\dagger - q \hat{b}_l^\dagger \hat{b}_k = \delta_{kl} \, , \tag{21}$$

where $\hat{b}_{\Delta\rho_{qvE}}$ and $\hat{b}^\dagger_{\Delta\rho_{qvE}}$ are the annihilation and creation operators which annihilate and create respectively micro-universes, associated with corresponding changes of the quantum vacuum energy density, and the cases $q = \pm 1$ correspond to bosons and fermions. The quantum Boltzmann statistics can be considered as an ideal mathematical tool to describe the pre-temporal state of quantum objects, able to explain the appearance of particles in terms of processes of creation and annihilation, namely of manifestation and de-manifestation of an opportune switching of elementary semi-local sub-quantum cells of the 3D DQV. In practice, it is as if space-time emerged from the actualization of these cells, and here the perspective is opened that the Higgs mechanism also has to do with these actualization processes. In this regard, in [35] it has been suggested that the scale of manifestation of particles is the chronon and that this scale can be considered as the result of a more fundamental physical switching of the cells of a de Sitter-Planck background.

In the light of these considerations, we define the following grand potential characterizing the multiverse as

$$\Omega(T, \mu) = \frac{\mathcal{E}'_s - \mathcal{E}_s}{q \ln\left(\frac{n_{s'}}{n_s}\right)} \sum_s \ln\left[1 + q \exp\left(\frac{\ln\left(\frac{n_{s'}}{n_s}\right)}{\mathcal{E}'_s - \mathcal{E}_s}(\mathcal{E}_s - \mu)\right)\right], \tag{22}$$

where μ is the chemical potential that determines whether we are in the eternal inflation regime or not, in the sense that when one is above this value universes are constantly being created, while when one is below they are not. In the light of equations (21) and (22), one can calculate the average cosmological constant in the ensemble of universes as

$$\langle \Lambda \rangle = \sum_s \frac{\Lambda_s \exp\left(\frac{\ln\left(\frac{n_{s'}}{n_s}\right)}{\mathcal{E}'_s - \mathcal{E}_s} (\mathcal{E}_s - \mu) \right)}{1 + q \exp\left(\frac{\ln\left(\frac{n_{s'}}{n_s}\right)}{\mathcal{E}'_s - \mathcal{E}_s} (\mathcal{E}_s - \mu) \right)}. \tag{23}$$

Here, if $\frac{\ln\left(\frac{n_{s'}}{n_s}\right)}{\mathcal{E}'_s - \mathcal{E}_s} (\mathcal{E}_s - \mu) \gg 1$, one obtains that the cosmological constant is a sum of suppressed exponential contributions and, therefore, one can explain the smallness of the cosmological constant without any anthropic assumption. In particular, one finds

$$\begin{aligned}\langle \Lambda \rangle &\approx \sum_s \Lambda_s \exp\left[\frac{\ln\left(\frac{n_{s'}}{n_s}\right)}{\mathcal{E}'_s - \mathcal{E}_s} (\mathcal{E}_s - \mu) \right] \\ &= \sum_s \Lambda_s \exp\left\{ \ln\left(\frac{n_{s'}}{n_s}\right) \left(\frac{4\pi\sqrt{3} m_{Pl}^3}{\sqrt{\frac{35Gc^2}{2\pi\hbar^4}\left(\frac{V_s}{c^2}\Delta\rho_{qvE}^{DEs}\right)^6}} - \mu \right) \right. \\ &\quad \left. \times \left[\frac{4\pi\sqrt{3} m_{Pl}^3}{\sqrt{\frac{35Gc^2}{2\pi\hbar^4}\left(\frac{V_{s'}}{c^2}\Delta\rho_{qvE}^{DEs'}\right)^6}} - \frac{4\pi\sqrt{3} m_{Pl}^3}{\sqrt{\frac{35Gc^2}{2\pi\hbar^4}\left(\frac{V_s}{c^2}\Delta\rho_{qvE}^{DEs}\right)^6}} \right]^{-1} \right\}. \end{aligned} \tag{24}$$

This formalism implies that universes characterized by negative cosmological constant should crunch immediately and do not give a contribution to the partition function. On the other hand, by using the static de-Sitter space – which turns out to be a recurring theme in discussions of the multiverse, specifically in the context of the holographic principle and thermodynamics [36–40] – universes with positive cosmological constants have a finite and time independent contribution. Here, it must be remarked that using the de-Sitter horizon, universes with planckian energy density are the least suppressed and will contribute the largest contributions to the average cosmological constant. Instead, as long as the temperature of the multiverse is subplanckian, which corresponds to a semi-classical regime, even universes with planckian energy density will not destroy the small cosmological constant. More generally, by assuming some distribution of cosmological constants in the different universes, the observed cosmological constant in our universe will constrain the temperature of the multiverse.

4 The Interacting Multiverse in the Three-Dimensional Quantum Vacuum Background: from the Generalized Wheeler-Dewitt Equation to the Cosmological Consequences

In this section, our aim is to provide a mathematical treatment of the interacting multiverse, inside a complete quantum picture. In this regard, we resort to the third quantization formalism developed in [41, 42] which seems to be the most appropriate syntax in order to describe the creation of pairs of interacting universes. In the third quantization formalism the wave function of the universe is promoted to the role of operator, and creation and annihilation operators are defined in a similar way to how is done in quantum field theory.

4.1 The generalized Wheeler-DeWitt equation in the three-dimensional quantum vacuum model and the consequences as regards the behaviour and features of each universe of the multiverse

In our approach, the fundamental, supra-universal background ruling the behaviour and the evolutionary possibilities of the universes composing the multiverse is represented by the 3D DQV characterized by a variable energy density. Each micro-universe can be described by a wave function Ψ, which is a function of the scale factor a of the universe and of the scalar field φ associated with the quantum vacuum energy density fluctuations as well as the polarization of the vacuum. Here, we consider that the evolution of the wave function Ψ of each micro-universe is ruled by a peculiar form of Wheeler-DeWitt (WDW) equation, intended as an equation of background, a real "equation of everything" where all the possibilities of the physical world are written in timeless form:

$$\ddot{\Psi} + \frac{V\Delta\rho_{qvE}}{ac^2\sqrt{\alpha}}\dot{\Psi} + \omega^2(a)\Psi = 0 \tag{25}$$

In equation (25) $\dot{\Psi} = \partial\Psi/\partial a$, α is the fine-structure constant, $\Delta\rho_{qvE}$ are the quantum vacuum energy density fluctuations corresponding to elementary RS processes of creation/annihilation of virtual particle/antiparticle pairs and ω is the mode frequency linked with the scale factor according to equation

$$\omega = \frac{1}{\hbar}\sqrt{a^4\Lambda - a^2 + \frac{V\Delta\rho_{qvE}}{\hbar l_p^2 a^2}}\,, \tag{26}$$

Λ being the cosmological constant, and that can be associated to an Hamiltonian of the form

$$H = \frac{1}{2}\left(-\frac{\partial^2}{\partial a^2} + \kappa a^2 - 2aH_\phi - a^4\frac{\Lambda}{3}\right), \tag{27}$$

where $\kappa = +1$, 0, -1 for spatially closed, flat and open universes respectively and H_ϕ is the Hamiltonian depending on the scale factor as a parameter. We call equation (25) as the "generalized WDW equation in the 3D DQV background". By invoking the third quantization formalism, the wave function Ψ of the universe satisfying WDW equation (25) is promoted to the role of operator, which can be expressed as

$$\hat{\Psi}_{\pm}=\frac{V\Delta\rho_{qvE}}{\sqrt{2\pi}\hbar l_p^2}\int d\rho\Bigg[\exp\left(\pm i\frac{V\Delta\rho_{qvE}}{\hbar l_p^2}\varphi\right)\Psi_{\Delta\rho_{qvE}}(a)\hat{b}_{\Delta\rho_{qvE}} + \exp\left(\mp i\frac{V\Delta\rho_{qvE}}{\hbar l_p^2}\varphi\right)\Psi^*_{\Delta\rho_{qvE}}(a)\hat{b}^\dagger_{\Delta\rho_{qvE}}\Bigg], \quad (28)$$

where $\hat{b}_{\Delta\rho_{qvE}}$ and $\hat{b}^\dagger_{\Delta\rho_{qvE}}$ are the annihilation and creation operators which annihilate and create respectively micro-universes, associated with corresponding changes of the quantum vacuum energy density, and satisfy the above mentioned "quantum Boltzmann statistics". The wave functions $\hat{\Psi}_+$ and $\hat{\Psi}_-$ are the two general operators which correspond to the two different possible branches of the universe. These two different branches of the universe, which are generated by the operators $\hat{\Psi}_+$ and $\hat{\Psi}_-$, are related by the time symmetry: by assuming that the universes are created in entangled pairs, they appear to be the same universe for any internal observer and have the property that before the big crunch singularities one branch can make a quantum transition to the other branch universe, appearing there as a newborn universe. In this way, a cyclic multiverse is generated where the evolutionary behaviour of each branch is determined and fixed by the specific quantum vacuum energy density fluctuations characterizing that region of the landscape.

Here, the production of a micro-universe can be associated to the creation of an information in a cell of the 3D DQV given by the following expression:

$$I = A/l_p^2\,, \quad (29)$$

where A is the area of the cell and $l_p \approx 10^{-33}$ cm is the Planck length. According to equation (29) and (30), a purely informational interpretation of the Planck scale emerges directly, which suggests interesting perspectives of unification between elementary particle physics and cosmology. In particular, by using a fruitful result of the transactional approach [43–45], the regime of ordinary Standard Model particles can be obtained by considering the chronon scale $A^3/l_p^2 \approx 10^{-13}$ cm. In the light of equation (29), we can therefore say that if the information created in a cell satisfies relation $I^3 l_p \approx 10^{-13}$ cm, then the corresponding micro-universe contains the ordinary Standard Model particles, which can be so seen as an emergent phenomenon from the processes of creation/annihilation of micro-universes associated with specific quantum vacuum energy density fluctuations and mathematically described by the wave function operator (28).

Let us analyse now in detail the physical meaning and consequences of the wave function operator (28) as regards the behaviour and features of each universe of the multiverse. In this regard, it must be remarked that, for $0 < a < 1/\sqrt{\Lambda}$, the wave function operator (28) may give origin to a classical universe, whilst for $a > 1/\sqrt{\Lambda}$ this wave function can produce the exponential decay of the Euclidean regime or the quantum transition. As one approaches the value $a = 0$, quantum effects become dominant and thus creation of cyclic universes in entangled pairs takes place. Furthermore, if quantum fluctuations of the wave function of the universe are considered, then, a minimum value $a_{\min}$ appears, below of which no real solution exists. In this classically forbidden region, double Euclidean instantons can be created giving rise, in the Lorentzian regime, to an entangled pair of universes whose quantum states turn out to be quantum-mechanically correlated. Moreover, if one invokes an antipodal-like symmetry for the time variables of the observers [46, 47], one obtains that, if according to an observer living in the universe with time variable t_1 his branch is the expanding branch and the preceding one is the contracting branch, according to the observer of the universe with time variable t_2 these two branches turn out to be the other way around, actually. Both observers are thus initially living in an expanding universe and the two branches can be combined to form an universe that is classically indistinguishable.

As regards the evolutionary behaviour of the scale factor of the two branches determined by the wave function operator (28), by following [48], one has

$$a\frac{da}{dt} \approx \mp\omega\left(a\right) \tag{30}$$

for the two signs given in equation (28). In other words, for the branch described by the wave function

$$\hat{\Psi}_{+} = \frac{V\Delta\rho_{qvE}}{\sqrt{2\pi}\hbar l_p^2}\int d\rho\Bigg[\exp\left(i\frac{V\Delta\rho_{qvE}}{\hbar l_p^2}\varphi\right)\Psi_{\Delta\rho_{qvE}}(a)\hat{b}_{\Delta\rho_{qvE}} + \exp\Big(-i\frac{V\Delta\rho_{qvE}}{\hbar l_p^2}\varphi\Big)\Psi^{*}_{\Delta\rho_{qvE}}(a)\hat{b}^{\dagger}_{\Delta\rho_{qvE}}\Bigg], \tag{31}$$

the scale factor obeys equation

$$a\frac{da}{dt} \approx -\omega\left(a\right); \tag{32}$$

for the branch described by the wave function

$$\hat{\Psi}_{-} = \frac{V\Delta\rho_{qvE}}{\sqrt{2\pi}\hbar l_p^2}\int d\rho\Bigg[\exp\left(-i\frac{V\Delta\rho_{qvE}}{\hbar l_p^2}\varphi\right)\Psi_{\Delta\rho_{qvE}}(a)\hat{b}_{\Delta\rho_{qvE}} + \exp\left(i\frac{V\Delta\rho_{qvE}}{\hbar l_p^2}\varphi\right)\Psi^{*}_{\Delta\rho_{qvE}}(a)\hat{b}^{\dagger}_{\Delta\rho_{qvE}}\Bigg], \tag{33}$$

the scale factor obeys equation

$$a\frac{da}{dt} \approx \omega\left(a\right). \tag{34}$$

Taking account of relation (26), equation (30) may be also formulated as follows:

$$\frac{da}{dt} = \pm\frac{1}{\hbar}\sqrt{a^2\Lambda - 1 + \frac{V\Delta\rho_{qvE}}{\hbar l_p^2 a^4}}. \tag{35}$$

On the basis of equations (30)-(35), in the multiverse generated by the variable energy density of the 3D DQV and described by the generalized WDW equation (26), we thus obtain two classical branches, one with a scale factor given by

$$a\left(t\right) = \frac{1}{\sqrt{\Lambda}}\sin\left[\left(\sqrt{\Lambda} + \frac{V\Delta\rho_{qvE}}{\hbar}\right)\left(t - t_0\right)\right] \tag{36}$$

and the other with a scale factor of

$$a\left(t\right) = \frac{1}{\sqrt{\Lambda}}\sin\left[\left(\sqrt{\Lambda} + \frac{V\Delta\rho_{qvE}}{\hbar}\right)\left(t_0 - t\right)\right]. \tag{37}$$

So, we can say that the real physical meaning of the wave function (28) becomes the following. Since here we deal with two different types of classical branches, respectively associated with the scale factors (36) and (37), the corresponding wave function operator (28) reads

$$\hat{\Psi}_{\pm} = \frac{V\Delta\rho_{qvE}}{\sqrt{2\pi}\hbar l_p^2}\int d\rho\left[\exp\left(\pm i\frac{V\Delta\rho_{qvE}}{\hbar l_p^2}\varphi\right)\Psi_{\Delta\rho_{qvE}}(a)\hat{b}_{\Delta\rho_{qvE_+}} + \exp\left(\mp i\frac{V\Delta\rho_{qvE}}{\hbar l_p^2}\varphi\right)\Psi^*_{\Delta\rho_{qvE}}(a)\hat{b}^\dagger_{\Delta\rho_{qvE_-}}\right], \tag{38}$$

where the operator $\hat{b}_{\Delta\rho_{qvE_+}}$ represents the annihilation operator of the branches of the universe given by (37), while $\hat{b}^\dagger_{\Delta\rho_{qvE_-}}$ represents the creation operator of the branches of the universe given by (36), both evaluated at the constant value $a = a_{\min}$. The branches are created in entangled pairs because of the quantum symmetry of the generalized WDW equation (25) with respect to the value $\pm\omega\left(a\right)$ of the classical branches, quantum-mechanically represented by (28).

4.2 A quantum field theory for the wave function of the multiverse in the curved minisuperspace

The vacuum state of the $\left(\hat{b}_{\Delta\rho_{qvE_\pm}}, \hat{b}^\dagger_{\Delta\rho_{qvE_\pm}}\right)$ turns out to be the ground state of the 3D DQV described by the Planck energy density (2) and cannot be considered as a stable vacuum state by virtue of the scale-factor dependence of the frequency $\omega(a)$ expressed by equation (26). Similarly to what occurs in a quantum field theory of a scalar field that propagates in a curved space-time, we can impose here the boundary condition for the proper representation for the vacuum state of the minisuperspace that has to be stable under the evolution of the universe along a geodesic of the minisuperspace. The minisuperspace that we are considering here is formed by the scale factor and the scalar field, φ, that represents the energy-matter content of the universe and, therefore, a geodesic of the minisuperspace is precisely the path given by the classical relation, $\varphi = \varphi(a)$. The boundary condition that the cosmological vacuum is stable along the geodesic of the minisuperspace means that it is stable under the classical evolution of the universes, i.e., once the multiverse is in the state of the invariant representation for some value $a_0 = a_{\min}$, then, it will remain in that state at any other value of the scale factor $a(t)$ along the evolution of any universe.

With these ingredients, we can now develop a quantum field theory for the wave function Ψ of the multiverse in the curved minisuperspace spanned by (a, φ) with a minisuperspace metric given by

$$G_{MN} = \begin{pmatrix} -a & 0 \\ 0 & a^3 \end{pmatrix}, \tag{39}$$

where M, N stand for $\{a, \varphi\}$. The scale factor a formally plays the role of the time variable and the matter field assumes the role of the spatial variable in the two-dimensional Lorentzian minisuperspace metric (39) ($a(t)$ can actually be seen as a time reparametrization). We can now follow the usual procedure of a quantum field theory for the scalar field $\Psi(a, \varphi)$ by considering the following action:

$$S = \int da d\varphi \left[-\frac{1}{2} \frac{ac^2\sqrt{\alpha}}{V\Delta\rho_{qvE}} \dot{\Psi}^2 + \frac{a\omega^2}{2} \Psi^2 \right], \tag{40}$$

which corresponds to the following Hamiltonian density

$$\mathcal{H} = -\frac{1}{2} \left[\frac{1}{a} P_\Psi^2 + a\omega^2 \Psi^2 \right], \tag{41}$$

where $P_\Psi = -a\dot{\Psi}$ is the momentum conjugated to the field Ψ. Now, by following [10], we consider the evolution of a set universes that are interacting each other where each internal observer sees its own Hamiltonian density. The total Hamiltonian density is thus expressed as

$$\mathcal{H} = \sum_{n=1}^{N} \mathcal{H}_n^{(0)} + \mathcal{H}_n^{(i)}, \tag{42}$$

where $\mathcal{H}_n^{(0)}$ is the unperturbed Hamiltonian density of the n-universe, given by equation (41), and $\mathcal{H}_n^{(i)}$ is the Hamiltonian density of the interaction for the n-universe, that here we consider as the simple quadratic interaction between next neighbour universes, namely

$$\mathcal{H}_n^{(i)} = \frac{a\lambda^2}{8}\left(\Psi_{n+1} - \Psi_n\right)^2 . \tag{43}$$

As regards the Hamiltonian density of the interaction for the n-universe, λ is a coupling function that depends only on the scale factor. Moreover, here, the Hamiltonian density of the interaction for the n-universe satisfies periodic boundary conditions so that $\Psi_{N+1} = \Psi_1$: this physically means that the universe is characterized by cyclic features, namely can be seen as a timeless phenomenon in dynamic equilibrium characterized by a cyclic transformation "space-matter-space-matter ...". In this picture, we can take into account, for the sake of simplicity, that the states of the generic universe may be described in terms of the normal modes by means of the Fourier transformation of Ψ and P_Ψ

$$\tilde{\Psi}_k = \frac{1}{\sqrt{N}}\sum_{n=1}^{N} \mathrm{e}^{-(2\pi ikn/N)}\Psi_n , \tag{44}$$

$$\tilde{P}_k = \frac{1}{\sqrt{N}}\sum_{n=1}^{N} \mathrm{e}^{-(2\pi ikn/N)}P_n . \tag{45}$$

As a consequence, the Hamiltonian density (42) becomes

$$\mathcal{H} = -\frac{1}{2}\sum_{k=1}^{N}\left(\frac{1}{a}\tilde{P}_k^2 + a\omega_k^2\tilde{\Psi}_k^2\right). \tag{46}$$

With this assumption, each micro-universe has a collective behaviour where the quantum states oscillate with frequency

$$\omega_k^2\left(a,\varphi\right) = \omega^2\left(a,\varphi\right) + \lambda^2\left(a\right)\sin^2\left(\frac{\pi k}{N}\right). \tag{47}$$

The oscillation of the wave function $\tilde{\Psi}_k$ results in an effective Wheeler-De Witt equation of the wave function of the k-universe in the $\tilde{\Psi}$ representation that appears as an isolated, non-interacting universe, of the form

$$\ddot{\tilde{\Psi}}_k + \frac{1}{a}\dot{\tilde{\Psi}}_k + \omega_k^2\left(a,\varphi\right)\tilde{\Psi}_k = 0. \tag{48}$$

Now, we observe that the frequencies (47) can be expressed as

$$\omega_k^2\left(a,\varphi\right) = \frac{1}{\hbar^2}\left(\tilde{H}_{1,k}^2 a^4 + H_0^2 a^4 - a^2\right), \tag{49}$$

where $H_0 = \Lambda$ and $\tilde{H}_{1,k}^2 = \dfrac{V\Delta\rho_{qvE}}{\hbar l_p^2 a^2}\tilde{V}_k\left(a,\varphi\right)$ and

$$\tilde{V}_k\left(a,\varphi\right) = V\left(\varphi\right) + \frac{\lambda^2\left(a\right)}{4\pi^2 a^4}\sin^2\left(\frac{\pi k}{N}\right), \tag{50}$$

where $V(\varphi)$ is a potential of the scalar field. By exploring the influence of the last term in equation (50) in the terms of the k-universe in a regime where the wave function of the k-universe can be approximately described by the semi-classical wave function

$$\Psi_k \approx \mathrm{e}^{\pm \frac{i}{\hbar} S_0(a)} \Delta_k (a, \varphi) \tag{51}$$

with S_0 being the action of the gravitational part alone with no interaction and the positive and negative signs in equation (51) correspond to the contracting and the expanding branches of Ψ_k, respectively, one obtains a Schrödinger-type equation for the scalar field of the form

$$-i\hbar \frac{\partial}{\partial t} \Delta_k = \frac{1}{2} \left(\frac{1}{a} \frac{\partial^2}{\partial \varphi^2} + a\tilde{V}_k (a, \varphi) \right) \Delta_k \tag{52}$$

and the following field equation for the scalar field:

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} + \frac{d\tilde{V}_k}{d\varphi} = 0\,, \tag{53}$$

where the dots stands for derivative with respect to the Friedmann time t, i.e. $\dot{\varphi} \equiv \dfrac{d\varphi}{dt}$. As regards the field equation (53), one can remark that, while the classical behaviour of the scalar field remains unaltered with respect the usual description because the last term in the potential $\tilde{V}_k$, given by equation (50), has no influence upon equation (53), instead, in the quantum regime, the extra term in the potential induces a modification in the vacuum state that has to be accounted for any vacuum decay process of the universe, resulting in the process of bubble formation and in a suggestive construction of the global structure of the 3D DQV.

In the interacting multiverse having the 3D DQV as fundamental underlying background, for a given value of the scale factor, the global effective value of the potential of the k-universe is given by a set of curves separated by k units, with $k = 0, \ldots, N/2$, which presents a landscape structure with N different vacua: $N-1$ false vacua states and a true vacuum state corresponding to a minimum of the potential $V(\varphi)$. Here one can consider, on one hand, the vacuum decay in an universe as a consequence of the multiverse interaction and, on the other hand, the vacuum decay into a real vacuum that corresponds to a single universe. As a consequence of the different possible behaviours of the quantum vacuum energy density fluctuations as well as of the QBS characterizing the micro-universes of the 3D DQV, the process of bubble formation and the global structure of a single universe is very rich, in the sense that small baby universes are created from quantum fluctuations of the background. At small values of the scale factor (i.e. values close to the Planck scale) the fluctuations of the scalar field and the effects of the interaction among the universes are dominant, so the newborn universes are expected to remain in normal modes with a high value of

k. In some universes, the effective value of the potential would be high enough to trigger inflation even if we assume the limit that is suggested by the Planck data [49]. Moreover, if one considers the expansion of the universe in the k-false vacuum, different processes of vacuum decay are expected to occur generating new bubbles of smaller and smaller false vacua until the bubbles are created in the true vacuum corresponding to the minimum of the potential of the scalar field. And here the process does not stop in the sense that the quantum fluctuations of the variable energy density imply the generation of a self-contained eternal process where the process of vacuum decay and bubble formation would take place continuously with false or true vacuum state that would supply new baby universes, with new systems of space-matter which are generated in dynamic equilibrium in a cyclic way.

4.3 The interaction between the universes of the three-dimensional quantum vacuum

In summary, in the model proposed in this paper, by using the third quantisation formalism and taking account of the QBS characterizing the micro-universes of the 3D DQV, our universe can be seen as a large parent universe propagating in a plasma of baby universes. As a first approximation, the quantum fluctuations of the space-time corresponds to particle-like bubbles of the 3D DQV of Planck size that originate from the parent universe and propagate therein. The third quantization formalism for the baby universes, combined with an opportune Lagrangian description for the matter fields, provides an accurate description of the coupling of these baby universes with the matter fields that propagate in the parent space-time. The effects of the baby universes can be measured by their influence on the observable properties of the parent universe, and here one can assume that, while the global behaviour of the space-time is classical, there are however fluctuations which can be seen as micro-universes (deriving from the virtual particles of the vacuum) propagating in the parent background. The total action which describes this baby–parent universe interaction can be expressed as

$$S_T = S_0(a,\varphi) + S_{\hat{b}_{\Delta\rho_{qvE}}}(\hat{\Psi}) + S_I(a,\varphi;\hat{\Psi})\,, \tag{54}$$

where $S_0(a,\varphi)$ is the Einstein-Hilbert action of the homogenous and isotropic parent space-time with scale factor a and matter field φ, $S_{\hat{b}_M}(\hat{\Psi})$ is the third quantised action of the baby universes, while $S_I(a,\varphi;\hat{\Psi})$ is the action of interaction,

$$S_I\left(a,\varphi;\hat{\Psi}\right) = \int dt\mathcal{N}\sum_i \mathcal{L}_i(t,\vec{x})\left(\hat{b}^{\dagger}_{\Delta\rho_{qvE}} + \hat{b}_{\Delta\rho_{qvE}}\right), \tag{55}$$

where the index i labels the different modes of the baby universe field (namely each i corresponds to each different species of baby universes), and $\mathcal{L}_i(t\vec{x})$ represents the insertion operator at the nucleation event in the sense that defines the space-time points of the parent universe in which the baby universes effectively nucleate.

Now, by following [50], one can introduce the following Hamilton-Jacobi equation describing the evolution of the action (54) of the individual universes in terms of the scale factor

$$\frac{\partial S_T}{\partial a} = \pm\sqrt{\frac{V\Delta\rho_{qvE}}{\hbar l_p^2 a^2}\left(V\left(\varphi\right)+\frac{\lambda^2(a)}{4\pi^2 a^4}\sin^2\left(\frac{\pi k}{N}\right)\right)a^4+\Lambda^2 a^4-a^2}. \quad (56)$$

Equation (56), by choosing the minus sign which corresponds to considering an expanding universe, leads to the following version of Friedmann equation for the background of a single universe within the interacting multiverse:

$$H^2_{\Delta\rho_{qvE}}\left(a\right) = H^2_{dS} + \frac{\lambda^2\left(a\right)}{4\pi^2 a^4}\sin^2\left(\frac{\pi k}{N}\right), \quad (57)$$

where H_{dS} is the quasi-constant Hubble parameter arising from the quasi-de Sitter evolution of this model universe caused by the constant scalar-field potential. Here one can easily remark that the interaction scheme between the universes composing the multiverse determines a modification of the effective form of the Friedmann equation in terms of a non-local correction. The value of $\frac{\lambda^2(a)}{4\pi^2 a^4}\sin^2\left(\frac{\pi k}{N}\right)$ is taken to be small in the sense that one can consider here small interactions that make the evolution of the universe slightly depart from the non-interacting case. Of course, as the scale factor approaches the Planck length, this approximation breaks down and one deals with a primordial era with large interactions giving rise to the creation and annihilation of universes (like in particle physics). However, in spite of the smallness of the constants in the correction term, it may induce a measurable departure from the evolution of a non-interacting universe.

It must be emphasized that the interaction between universes and the generation of new bubbles are expected to be dominant only for small length scales of the parent background, where quantum effects of the variable energy density fluctuations are significant. These newborn bubbles have the potential to generate new bubbles in a self-reproducing process. The vacuum decay between the quantum states of two or more universes is expected to be cut off for large values of their scale factors. However, differently from other cosmological scenarios where a fine-tuning is required [51], in our approach the fine-tuning is not inserted ad hoc but is automatically embedded in the formalism, in other words appears as a direct consequence of the dynamic equilibrium between energy density of matter and energy density of space as well as of the action of dark energy as an interplay of more fundamental quantum vacuum energy density fluctuations.

Finally, as regards the emergence of time, the scenario predicted by our model is the following. When the multiverse is in the vacuum state, which is quantum-mechanically described by the ground state of the invariant representation of the minisuperspace, which corresponds to the Planck energy density (2), its entropy is constantly zero, thus expressing a stable system, which corresponds to a multiverse where there is no arrow of time. However, fluctuations of the quantum vacuum energy density generate the appearance of a physical arrow of time which therefore exists only as an emerging parameter which measures the numerical order of the dynamics characterizing the systems of each universe. Moreover, this arrow of time can be associated to the entropy of entanglement of each single universe, which not only is not zero but it evolves with respect to the value of the scale factor, and provides a correlation between the physical and the mathematical arrows of time in each individual universe, as it corresponds to the point of view of an internal observer who does not see the rest of the multiverse.

5 About Galactic Dynamics and Dark Matter in the Multiverse Scenario of the Three-Dimensional Quantum Vacuum

As regards the cosmological constant problem, in the approach based on the generalized WDW equation (25), universes with different values of the cosmological constant are possible. In reference to the cosmological constant, in our model, the Friedmann equations can be formulated as follows

$$\frac{\dot{a}^2(t)}{a^2(t)}+\frac{kc^2}{R_0^2a^2(t)}=\frac{8\pi G_N}{3}\frac{\Delta\rho_{qvE}}{c^2}\left[k+\frac{\hbar}{2}\left(1+\beta l_p^2\frac{\Delta\rho_{qvE}^2V^2}{\hbar^2c^2}\right)\right]+\frac{\Lambda c^2}{3} \quad (58)$$

and

$$\frac{\ddot{a}(t)}{a(t)}=-\frac{4\pi G_N}{3}\left\{\frac{3p}{c^2}+\frac{\Delta\rho_{qvE}}{c^2}\left[k+\frac{\hbar}{2}\left(1+\beta l_p^2\frac{\Delta\rho_{qvE}^2V^2}{\hbar^2c^2}\right)\right]\right\}, \quad (59)$$

where "dot" denotes the derivative with respect to time t, $\dot{a}(t)=\frac{da(t)}{dt}$, $\rho=\frac{\Delta\rho_{qvE}}{c^2}\left[k+\frac{\hbar}{2}\left(1+\beta l_p^2\frac{\Delta\rho_{qvE}^2V^2}{\hbar^2c^2}\right)\right]$ is the mass density (V being the volume of the region into consideration), p is the pressure, k denotes the curvature signature, $R(t)$ is the proper distance at epoch t, $R_0=R(t_0)$ is the distance at the reference time t_0 (that can be also interpreted as the radius of the Universe now).

As regards equation (59), if we consider the identity $\frac{\ddot{a}(t)}{a(t)}=\frac{\ddot{R}(t)}{R(t)}$ and we multiply this equation by the product $mR(t)$, we find a Newton-type equation of motion of the form

$$m\ddot{R}(t)=-G\frac{m\frac{\Delta\rho_{qvE}V}{c^2}\left[k+\frac{\hbar}{2}\left(1+\beta l_p^2\frac{\Delta\rho_{qvE}^2V^2}{\hbar^2c^2}\right)\right]}{R^2(t)}+m\frac{\Lambda c^2}{3}R(t) \quad (60)$$

for the point mass m in a sphere of radius $R(t)$. Equation (60) turns out to be completely equivalent to equation (59), if one divides (60) by m and then utilizes the identity $R(t) = a(t)R_0$. It must be remarked that the two terms on right-hand side of equation (60) play the same role as forces in Newton's equation of motion. In particular, the third term, namely $m\frac{\Lambda c^2}{3}R(t)$, under the constraint $\Lambda > 0$ represents a repulsive force which grows in intensity with increasing $a(t)$, while under the constraint $\Lambda < 0$ it turns out to be an attractive force.

Moreover, as a consequence of the oscillatory behaviour of the cosmological constant $\Lambda(t)$, the acceleration $\ddot{R}(t) = \ddot{a}(t)R_0$ increases or decreases depending on whether the corresponding $\Lambda(t)$ is positive or negative and therefore the radius $R(t) = a(t)R_0$ of the sphere increases slower or faster, in other words our sphere is vibrating. Finally, always in this phase of the evolution, by inserting the oscillatory function $\Lambda(t)$ inside equation (58), also the Hubble parameter $H(t) = \frac{\dot{a}(t)}{a(t)}$ turns out to be oscillatory at this time region.

Moreover, it is interesting to remark that the Newton-type equation of motion (50) deriving from the Friedmann equation (59) can open suggestive perspectives in the explanation of the galactic dynamics. The total force appearing in the right-hand side of equation (50) can be seen as the sum of the following three terms:

$$F_1 = -G\frac{m\frac{\Delta\rho_{qvE}V}{c^2}k}{R^2(t)}, \tag{61}$$

$$F_2 = -G\frac{m\frac{\Delta\rho_{qvE}V}{c^2}\frac{\hbar}{2}\left(k + \beta l_p^2\frac{\Delta\rho_{qvE}^2V^2}{\hbar^2c^2}\right)}{R^2(t)}, \tag{62}$$

$$F_3 = m\frac{\Lambda c^2}{3}R(t). \tag{63}$$

Now, under the constraint $G\frac{\frac{\Delta\rho_{qvE}V}{c^2}k}{R^2(t)} \gg \frac{\Lambda c^2}{3}R(t)$, at the zero-th order, F_2 and F_3 are negligible and thus one obtains an acceleration $a_N = G\frac{\Delta\rho_{qvE}V}{c^2}\Big/r^2$, which is the familiar Newtonian value for the acceleration due to the source $\frac{\Delta\rho_{qvE}V}{c^2}$. Instead, for $G\frac{\frac{\Delta\rho_{qvE}V}{c^2}k}{R^2(t)} \ll \frac{\Lambda c^2}{3}R(t)$, F_1 turns out to be negligible and therefore one finds

$$F \approx m\frac{a^2}{2\left[\frac{\Lambda c^2}{3}R(t) - G\frac{m\frac{\Delta\rho_{qvE}V}{c^2}\frac{\hbar}{2}\left(\beta l_p^2\frac{\Delta\rho_{qvE}^2V^2}{\hbar^2c^2}\right)}{R^2(t)}\right]} \tag{64}$$

and thus the terminal velocity v of the mass m in a circular motion with radius r can be obtained from relation

$$m\frac{a_N^2}{2\left[\frac{\Lambda c^2}{3}R(t)-G\frac{m\frac{\Delta\rho_{qvE}V}{c^2}\frac{\hbar}{2}\left(\beta l_p^2\frac{\Delta\rho_{qvE}^2V^2}{\hbar^2c^2}\right)}{R^2(t)}\right]}=\frac{mv^2}{r}\,. \tag{65}$$

By following [50], in the small acceleration regime, the observed flat galactic rotation curves require that in this regime the acceleration is given by expression

$$a\approx\left(2a_N\left[\frac{\Lambda c^2}{3}R(t)-G\frac{m\frac{\Delta\rho_{qvE}V}{c^2}\frac{\hbar}{2}\left(\beta l_p^2\frac{\Delta\rho_{qvE}^2V^2}{\hbar^2c^2}\right)}{R^2(t)}\right]^3/\pi\right)^{1/4}\,. \tag{66}$$

As a consequence, one then finds that, in this regime, the force can be expressed through relation

$$F\approx m\sqrt{a_N a_c}\,, \tag{67}$$

where

$$a_c=\frac{1}{2\pi}\left[\frac{\Lambda c^2}{3}R(t)-G\frac{m\frac{\Delta\rho_{qvE}V}{c^2}\frac{\hbar}{2}\left(\beta l_p^2\frac{\Delta\rho_{qvE}^2V^2}{\hbar^2c^2}\right)}{R^2(t)}\right]. \tag{68}$$

Equation (68) can be compared with the corresponding critical acceleration parameter originally discovered by Milgrom [52–54] in the modified Newtonian dynamics scaling, given by expression

$$a_c=\frac{a_0}{2\pi}\,, \tag{69}$$

which yields $a_c = 10^{-8}$ cm/s^2 by positing $a_0 = cH$. Therefore, in our approach, the perspective is opened to explain the corrected magnitude for the critical galactic acceleration parameter in terms of the cosmological scenario of interacting micro-universes generated by the timeless 3D DQV, described by the primeval wave function (28). Furthermore, here the modified Newtonian dynamics can itself be seen as a phenomenological consequence of more fundamental properties of the 3D DQV background described by the generalized WDW equation. And if, in the light of the results obtained by Ng in [55], in modified Newtonian gravity dark matter emerges as the manifestation of a Mondian force, at the galactic scale, of the type

$$2\pi k_B\tilde{T}=G\tilde{M}/r^2\,, \tag{70}$$

where $\tilde{T}$ is the net Unruh-Hawking temperature, $\tilde{M} = M + M_d$ is the total mass contained in a volume $V = \frac{4}{3}\pi r^3$, and

$$M_d\approx\frac{1}{\pi}\left(\frac{a_0}{a}\right)^2 M\,, \tag{71}$$

in a similar way, in our approach of a 3D DQV background characterized by a variable energy density, on the basis of equations (60)-(69) it follows that dark matter can be seen as a manifestation of the total force given by the sum of the forces (61), (62) and (63), thus yielding

$$M_d \approx \frac{1}{\pi}\left(\frac{\frac{\Lambda c^2}{3}R(t) - G\frac{m\frac{\Delta\rho_{qvE}V}{c^2}\frac{\hbar}{2}\left(\beta l_p^2\frac{\Delta\rho_{qvE}^2 V^2}{\hbar^2 c^2}\right)}{R^2(t)}}{a}\right)^2 M\,. \qquad (72)$$

In the light of equation (72), we can say that the ultimate non-local texture of the 3D DQV background can be seen as a fundamental source of the origin of dark matter, in other words that dark matter is a manifestation of the ultimate properties of the geometry of the 3D DQV background.

6 Conclusions

The idea of multiverse consisting of a multitude of universes that have different properties, namely different physical constants and laws, has been formulated into a flourish mathematical shape within the framework of the superstring landscape as well as eternal inflationary model. Despite it provides an answer to a series of open problems in theoretical physics, such as the problem of the beginning of the universe or the smallness of the cosmological constant, as well as the more general fine-tuning of physical constants, the concept of multiverse which emerges from superstring theory and chaotic eternal inflation only partially solves these problems, indeed establishing a reformulation of the question. For example, as regards the fine-tuning problem, the multiverse in the current theories invokes a distribution of values of certain fundamental constants among the different possible universes, without being able to explain why we live precisely in an universe with the observed values in an independent way from some form of the anthropic principle [56].

The model, proposed in this paper, of a 3D DQV defined by a variable energy density as a supra-universal background ruling the evolutionary behaviour, can go a step ahead as regards the idea of multiverse and the perspectives of solution of the above mentioned problems. The model of the interacting multiverse, which is based on the generalized WDW equation (25) and the corresponding wave function operator (28), in the picture of a Quantum Boltzmann Statistics, allow us to find new keys of explanation of the different features of the parallel universes in a more fundamental picture with respect to superstring theory and eternal inflationary model, in the sense that here the different evolutionary possibilities, such as expansion and contraction, as well as the different values of the cosmological constant of the parallel universes, are directly fixed by (and thus can be seen as the direct consequence of) the different specific behaviours of the elementary energetic fluctuations of the 3D DQV. Because of the dependence of the rapidity of variation of the scale factor with the frequency mode of the generic universe, in the multiverse one deals with two kinds of classical branches, which are created in entangled pairs. In the model of the interacting multiverse where the 3D DQV is the fundamental underlying background, the different possible behaviours of the quantum vacuum energy density fluctuations as well as the QBS characterizing the background, imply that the process of bubble formation is very rich, in the sense that small baby universes are generated by the elementary quantum fluctuations of the background. At small values of the scale factor (i.e. values close to the Planck scale) the fluctuations of the scalar field and the effects of the interaction among the universes are dominant and thus one deals with the creation of cyclic universes in entangled pairs, in other words our universe can be seen as a large parent universe propagating in a plasma of baby universes. In this picture, the process of creation of these newborn bubbles can be considered as an eternal process, which gives origin in a cyclic way to new systems of space and matter in dynamic equilibrium.

Our cosmological approach has the merit to explain the corrected magnitude for the critical galactic acceleration parameter originally invoked by Milgrom in the modified Newtonian dynamics in terms of the interacting universes generated by the timeless 3D DQV, described by the primeval wave function operator (28). Moreover, it leads to a suggestive interpretation of dark matter as a manifestation of the ultimate properties of the geometry of the 3D DQV background.

The next step regards the possibility to test the ideas of the model of the interacting multiverse in the 3D DQV background by observations. The possibilities of verifiability of such an idea can be associated to the entanglement, to the quantum correlations between some classically disconnected universes, that can have some influence on observational quantities in our universe or in other universes of the ensemble. As regards the exploration of possible experimental test of the model of the interacting multiverse in the 3D DQV, further research will give you more information.

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