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Mass Spectra of Heavy Diquarks

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Abstract. The aim of the present work is to estimate the masses of the ground and radially excited state of heavy diquark systems. For this we use the non relativistic model of diquarks which are composed of quark–quark two body potential and spin–spin interaction. We computed the masses of the ground and excited states of *bb*, *bc* and *cc* masses. The results are compared with some previous works.

KEY WORDS: *bb* diquark, *bc* diquark, *cc* diquark, Strongly Coupled Quark Gluon Plasma, Quarkonium

1 Introduction

The hadronic system undergoes a phase transition from hadronic state to a deconfined state of quarks and gluons at high density and/or temperature [1]. The plasmonic state of deconfined quarks and gluons with respect to color charges is called Quark Gluon Plasma (QGP) [2]. QGP near and above Tc is proposed as strongly coupled coulombic plasma of color charges which is called Strongly Coupled Quark Gluon Plasma (SCQGP) [3]. The SCQGP is in several ways similar to some kind of electromagnetic plasma containing electric charges [4]. In [5], it is proposed that near and above T_c there are multiple bound states of quasi-particles which are very important for the thermodynamics of the QGP. At $T \geq T_c$, it has been argued that there is no color confinement and there is Coulomb-like interaction at small distances with a Debye type screening at large distances [6].

The deconfined quarks and gluons pair up non-perturbatively in order to lower the energy of the system. This may lead to the formation of a system of two quarks called diquark [7] or multi quark system consisting of more number of quarks. These multi-quark systems can be studied using different models such as the MIT bag model [8] and the Potential model [9]. Out of all possible multi-quark states, the study of diquarks is more important in the context of QGP formed in RHIC experiments [10]. In 1964 GellMann first mentioned the possibility of diquarks. A diquark is any system of two quarks considered collectively. The idea of diquarks has been used in hadron physics for describing many properties of hadrons [7]. This study is also important in the context of astrophysical

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conditions [11], especially in cosmological contexts involving early phases of the evolution of the universe [12]. The question about the existence of diquarks as quasi-bound states in cold quark matter was emerged in [10], in which the authors have claimed that the quark-diquark matter was energetically more favorable than the free quark matter. The idea of the possible existence of quasi-bound states of quarks and gluons including diquarks with the temperature in the range $T_c \leq T \leq 4T_c$ was put forward by Shuryak et al. [5]. In [13] it was shown that the Λ_c/D^0 ratio could be enhanced by a factor of 4–8 than the case without diquarks in quark-qluon plasma. Diquark correlations play an important role inside baryons. Two quarks bound in a color anti triplet configuration can couple with a single quark to form a color singlet baryon. Recent studies of baryons as bound states of a quark and a confined diquark are done [14]. Some lattice simulations also indicate the existence of correlations in the diquark channel. A diquark in its ground state has positive poarity and may be a scalar (Spin 0) or an axial vector (Spin 1). It has been pointed out that the production of scalar diquarks is much larger than the vector diquarks. Evidence for scalar diquarks has been confirmed by lattice QCD simulation [15].

This paper is organized as follows: In Section 2, we give the details of potential and the solution of Schrödinger equation. In Section 3 the binding energy of heavy diquarks is studied. In Section 4, the masses of diquarks are calculated. Section 5 gives an idea about the dissociation temperature of these heavy diquarks. Section 6 is the conclusion and summary of the results obtained.

2 Exact Solution of the N-dimensional Schrödinger Equation with the Two Body Potential in SCQGP at Finite Temperature

The 3-dimensional non-relativistic radial Schrödinger equation for two particles [16, 17] interacting via the two-body potential is given by

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2} + 2\mu(E_{nl} - V(r))\right]\Psi(r) = 0\,, \tag{1}$$

where n and l are the principal quantum number and angular quantum number respectively. The reduced mass of two particles μ is defined as

$$\mu = \frac{M_{q1}M_{q2}}{M_q 1 + M_{q2}}\,,$$

where $V(r)$ is the two-body potential defined in [18]

$$V(r) = \frac{Q}{4\pi r}\Big(\cos(k_i r) + \frac{1}{\sqrt{3}}\sin(k_i r)\Big)e^{-k_r r}\,, \tag{2}$$

where $k_i = k_s/2$ and $k_r = \sqrt{3}k_s/2$ with $Q = q_1 q_2$, where $k_s = \sqrt{\dfrac{M_q\omega_2^2}{g_c\left(\frac{\partial P}{\partial n_f}\right)}}$ is the inverse Thomas-Fermi screening length with $\omega p^2 = \frac{4}{3}\alpha_s n_{0f}/M_q$.

The temperature dependence is through the running coupling constant in SC-QGP which is given by [3]

$$\alpha_s(T) = \frac{6\pi}{(33-2n_f)\ln(T/\Lambda_T)}\Big(1 - \frac{3(153-19n_f)}{(33-2n_f)^2}\frac{\ln(2\ln(T/\Lambda_T))}{\ln(T/\Lambda_T)}\Big). \quad (3)$$

Here $g_c$indicates the color degrees of freedom, n_f gives the number density corresponding to the particular flavor, M_q is the quark mass and $\frac{\partial P}{\partial n_f}$ is given by [[18].

Using Nikiforov-Uvarov (NU) method, the energy eigen values at finite temperature can be written as [19]

$$E^N_{nl} = B + \frac{3C}{\delta} + \frac{6D}{\delta^2} - \frac{2\mu(-A+3C/\delta^2+8D/\delta^3)^2}{\Big((2n+1)\pm\sqrt{1+8\mu C/\delta^3+24\mu D/\delta^4+4l(l+1)}\Big)^2}, \quad (4)$$

where

$$A = \frac{Q}{4\pi},$$
$$B = \frac{Q}{4\pi}\Big(-k_r + \frac{k_i}{\sqrt{3}}\Big),$$
$$C = \frac{Q}{4\pi}\Big(\frac{k_r^2}{2} - \frac{k_i k_r}{\sqrt{3}} - \frac{k_i^2}{2}\Big),$$
$$D = \frac{Q}{4\pi}\Big(-\frac{k_r^3}{6} + \frac{k_2 k_r^2}{2\sqrt{3}} - \frac{k_i^2 k_r}{2} - \frac{k_i^3}{6\sqrt{3}}\Big).$$

Equation (4) gives the expression for the binding energy of the bound state of quarks. The radial wave function of Eq. (1) is given by

$$R^N_{nl}(r) = C_{nl} r^{-b/\sqrt{2a}-1} e^{\sqrt{2a}\,r}\Big(-r^2\frac{d}{dr}\Big)^n\Big(r^{-2n+2b/\sqrt{2a}}\,e^{-2\sqrt{2a}\,r}\Big), \quad (5)$$

where, $a = 2\mu D_1$ and $b = 2\mu D_2$ with

$$D_1 = -\Big[E - B_{\frac{3C}{\delta}} - \frac{6D}{\delta^2}\Big], \qquad D_2 = -A + \frac{3C}{\delta^2} + \frac{8D}{\delta^3}.$$

3 Binding Energy of *cc*, *bb* and *bc* Diquarks

The binding energy of bound state of quarks is calculated in the 3-dimensional space for any state at finite temperature using the NU method which is given by Eq. (4).

Here $M_{q1} = M_{q2} = M_b$ for *bb* diquark and $M_{q1} = M_b$ and $M_{q2} = M_c$ for *bc* diquark and $M_{q1} = M_{q2} = M_c$ for *cc* diquark. The variation of binding energy with temperature for different states of *bb*, *bc* and *cc* diquarks is shown in Figures 1, 2 and 3.

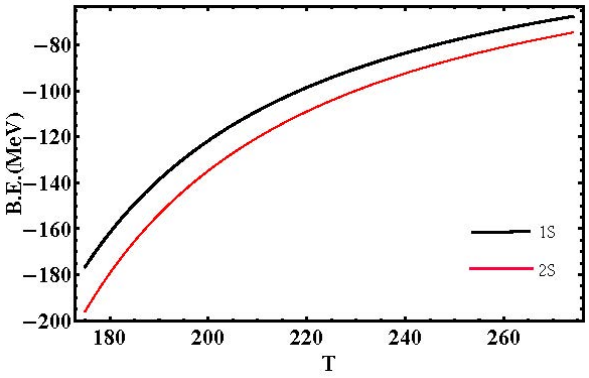

Figure 1. Variation of *bb* binding energy on temperature T.

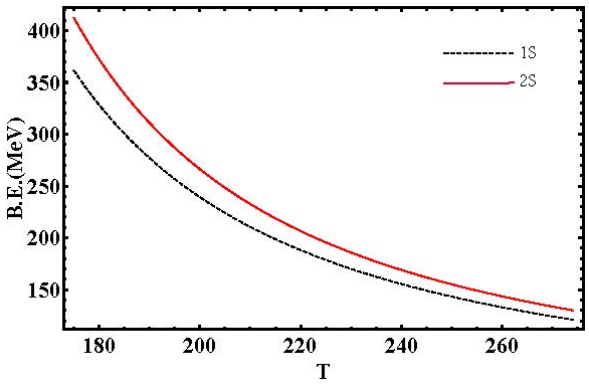

Figure 2. Variation of *bc* binding energy on temperature T.

4 Mass Spectra of Quarkonia

For calculating the mass of diquarks following relations are used [20].

$$M_{bb} = 2M_b + E_{nl} + \langle V_{ss} \rangle \,, \tag{6}$$

$$M_{cc} = 2M_c + E_{nl} + \langle V_{ss} \rangle \,, \tag{7}$$

$$M_{bc} = M_b + M_c + E_{nl} + \langle V_{ss} \rangle \,. \tag{8}$$

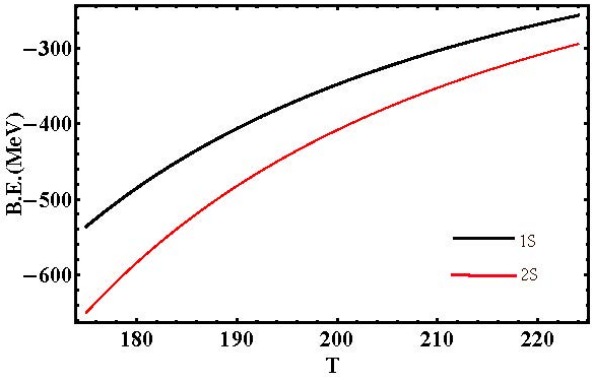

Figure 3. Variation of cc binding energy on temperature T.

Here $\langle V_{ss}\rangle$is the spin dependent term added to the potential $V(r)$ which is given by [21]. This term is related to the S-wave splittings. When $l \neq 0$, this term has no effect.

5 Dissociation Temperature of *bb*, *bc* and *cc* Ddiquarks

The dissociation temperature is defined as the smallest temperature where no resonance structures can be observed. When the binding energy of a particular bound state falls below the temperature, that state is weakly bound and can be destroyed by thermal fluctuations. The binding energy need not be zero for the dissociation of a bound state. Here we use the condition $E_{\text{bin}} = T$(where E_{bin} is the binding energy of a particular state) to find out the dissociation temperatures [28] for the different states of *bb*, *bc* and *cc* diquarks. The dissociation temperatures for different states are summarized in Table 4.

6 Results and Discussions

At finite temperature the behavior of *bb*, *bc* and *cc* diquarks is discussed. In Figures 1, 2 and 3, the binding energy of *bb*, *bc* and *cc* diquarks is plotted as a function of temperature.

Table 1. The mass spectra of *bb* states (in GeV) using $M_b = 4.812$ GeV

$n^{2S+1}l_J$	Present work	[21]	[22]	[23]
1^3S_1	9.709	9.792	9.72	9.74
2^3S_1	10.048	10.011	10.01	10.02

Here the critical temperature is taken as 175 MeV. As the temperature increases, the binding becomes weaker. In Tables 1, 2 and 3 the mass spectra of *bb*, *bc* and *cc* diquarks are calculated in comparison with some other theoretical works. Here the parameters are chosen according to [21] for good comparison. In Table 4, the dissociation temperatures for *bb*, *bc* and *cc* mesons are summarized. We have found that both *bc* and *cc* diquarks can dissociate at temperatures above the critical temperature ($T_c < T_D < 2T_c$). But the *bb* diquark can dissociate nearly at T_c.

The diquarks can be described as elementary excitations stimulating the many-body interactions and behave like scalar bosons [29]. Two quarks should be antisymmetric in color so that a total color singlet state is obtained. Recent lattice studies have found that the bound states can persist to at least $T = 2T_c$ [2]. Our potential also supports this condition. In general, diquark masses have contributions from the constituent masses, the confining and spin-dependent part of the quark-quark interaction [30]. These contributions are additive in a simple potential model [31]. The spin-dependent contribution may include a contribution from flavor-spin and color-spin dependent interaction terms.

Table 2. The mass spectra of *bc* states (in GeV) using $M_b = 4.812$ GeV and $M_c = 1.4$ GeV

$n^{2S+1}l_J$	Present work	[24]	[25]	[26]
1^3S_1	6.432	6.38	6.50	6.562
2^3S_1	6.753	6.66		6.883

Table 3. The mass spectra of *cc* states (in GeV) using $M_c = 1.4$ GeV

$n^{2S+1}l_J$	Present work	[21]	[27]	[22]
1^3S_1	3.241	3.114	3.133	3.13
2^3S_1	3.453	3.443	3.456	3.47

Table 4. The dissociation temperature (T_D) with $T_c = 175$ MeV for the $b\bar{c}$ meson states using $M_c = 1710$ MeV and $M_b = 5050$ MeV

State	$1S$	$2S$
bb	$1.005T_c$	$1.028T_c$
bc	$1.220T_c$	$1.232T_c$
cc	$1.325T_c$	$1.382T_c$

7 Conclusions

In the present work the binding energy and mass spectra of *bb*, *bc* and *cc* diquarks are obtained by solving the radial Schrödinger equation using NU method. We have investigated the properties of *bb*, *bc* and *cc* diquarks at finite temperature in strongly coupled quark-gluon plasma. We have estimated the binding energy and plotted as a function of temperature. The dissociation temperatures of various states of *bb*, *bc* and *cc* diquarkss are reported and it can be concluded that they can survive above the temperature for deconfinement of the medium and can give important information on the properties of the hot medium.

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